Astroparticle Physics
Instructor: A.M. van den Berg

You don't have to use separate sheets for every question.
Write your name and S number on every sheet
There are **4 questions** with a total number of marks: 33

WRITE CLEARLY

(1) (Total 6 marks)
The production of photons in a medium is an important mechanism to detect high-energy particles.

(a) (3 marks)
Cherenkov radiation is one of these mechanisms. What conditions are required to create Cherenkov radiation?

(b) (3 marks)
Name two other mechanisms which can be responsible for the creation of photons from high-energy particles.

(2) (Total 15 marks)
The Friedmann equation can be deduced from Newtonian mechanics based on a summation of energies. The Friedmann equation is given as:

$$\frac{\dot{R}^2}{R^2} = H^2 = \frac{8\pi}{3} G\rho - \frac{kc^2}{R^2}$$

In addition we can define the so-called critical energy density ρ_c as:

$$\rho_c = \frac{3H^2}{8\pi G}$$

(a) (3 marks)
In addition to the assumption that we can use Newtonian mechanics to derive the Friedmann equation, it holds only when a specific condition is satisfied. Under what name is this condition known and what does it mean?

(b) (3 marks)
Based on Newtonian mechanics, the three important types of energy are: total energy, kinetic energy, and potential energy. Identify these different energies with each of the terms in the Friedmann equation listed above.

(c) (3 marks)
What is the consequence for the Universe in case the energy density ρ equals the critical energy density in terms of the total energy contained in the Universe?

(d) (3 marks)
In the radiation-dominated Universe the energy density ρ_r scales as T^4, where T is the temperature of the photons. Work out an approximation for the Hubble constant as a function of the temperature and of the time t, where $t = 0$ at the Big Bang.

(e) (3 marks)
Which other energy densities play an important role in the development of the Universe?

(3) (Total 6 marks)

(a) (3 marks)
Use the relativistic equations underneath to find an expression for the Doppler shift.

$$p'_x = \gamma(p_x - \beta E)$$
$$p'_y = p_y$$
$$p'_z = p_z$$
$$E' = \gamma(E - \beta p_x)$$

(b) (3 marks)
The redshift is given as:

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}}$$

Show that for small values of β, that $z = \beta$.

(4) (Total 6 marks)
During the first 5 minutes of the early Universe, several chemical elements have been produced. For instance after these 5 minutes the mass fraction of helium is according to measurements and calculations equal to about 25%.

(a) (3 marks)
Which parameters played a decisive role in the production of ^{4}He to these amounts?

(b) (3 marks)
Name two other chemical elements which have been produced during these first 5 minutes.

1a charged particle
velocity $\beta > 1/n$, where n is index of refraction
medium is dielectric (can be polarized)

1b scintillation, fluorescence, π^0 decay, bremsstrahlung

2a cosmological principle
universe is homogeneous and isotropic

2b kinetic energy, gravitational energy, total energy

$$\frac{\dot{R}^2}{R^2} = H^2 = \frac{8\pi}{3} G\rho - \frac{kc^2}{R^2}$$

multiply with $\frac{1}{2}mR^2$ to get:

$$\frac{1}{2}m\dot{R}^2 = \frac{4\pi}{3} G\rho R^2 - \frac{1}{2}mkc^2$$

$\frac{1}{2}m\dot{R}^2$ = kinetic energy

$\frac{4\pi}{3}G\rho R^2$ = gravitational energy

$E = -\frac{mc^2k}{2}$ is total energy

2c) $\rho = \rho_c$ gives $k = 0$, no curvature

2d Radiation dominated $\rho_r \sim R^{-4}$

2d. $H^2 = \frac{\dot{R}^2}{R^2} = \frac{8\pi}{3} G\rho_r - \frac{kc^2}{R^2}$

Radiation dominated was R small; neglect terms $\frac{1}{R^2}$ wrt $\frac{1}{R^4}$

$$H^2 = \frac{\dot{R}^2}{R^2} \propto \frac{8\pi}{3} G \frac{1}{R^4}$$

$$H \sim \frac{1}{R^2}$$

Because $\rho_r \sim R^{-4}$ and $\rho_r \propto T^4$ (Stefan's law)

$$T \sim \frac{1}{R}$$

Thus $\boxed{H \propto T^2}$

For the dependance of H on time t we need to solve differential equation

$$\rho_r \propto R^{-4} \Rightarrow \frac{\dot{\rho}_r}{\rho_r} = -4\frac{\dot{R}}{R} = -4\left(\frac{8\pi}{3}G\rho_r\right)^{1/2}$$

$$= -4\alpha\rho_r^{1/2}$$

$$\frac{d\rho_r}{\rho_r^{3/2}} = -4\alpha\, dt$$

$$\rho_r^{-1/2} = 2\alpha t \qquad \rho_r = \frac{1}{4\alpha^2 t^2}$$

$$H = \frac{\dot{R}}{R} \propto \rho_r^{1/2} \propto \frac{1}{t} \Rightarrow \boxed{H \propto \frac{1}{t}}$$

2 e dark energy (or vacuum energy) density
matter density

3a $E' = h\nu' = \gamma(h\nu - \beta h\nu\cos\theta)$

$= \gamma h\nu(1-\beta\cos\theta)$

Assume $\theta = 0$ $\quad h\nu' = \gamma h\nu(1-\beta)$

$\nu' = \gamma(1-\beta)\nu$

$$\lambda' = \frac{\lambda}{\gamma(1-\beta)}$$

3 b $$\frac{1}{\gamma(1-\beta)} = \frac{(1-\beta^2)^{1/2}}{(1-\beta)} = \frac{(1-\beta)^{1/2}(1+\beta)^{1/2}}{1-\beta}$$

$$= \frac{(1+\beta)^{1/2}}{(1-\beta)^{1/2}} \approx \left(1+\frac{1}{2}\beta\right)\left(1+\frac{1}{2}\beta\right)$$

$$\approx 1+\beta+\dots$$

$$z = \frac{\Delta\lambda}{\lambda} = \frac{\lambda_{obs}-\lambda_{em}}{\lambda_{em}} \simeq \frac{(1+\beta)\lambda_{em}-\lambda_{em}}{\lambda_{em}}$$

$$\simeq \beta$$

4a 1) temperature drop compared to expansion rate
2) life time of neutron
3) mass difference between n and p
4) number of neutrinos / m^3
5) number of photons / m^3
6) ...

4b Li, H, Be

note that ^1H ^2H (deuteron) and ^3H are different isotopes of only one element: hydrogen!